

# Local similarity solutions of free convective heat transfer from a vertical plate to non-Newtonian power law fluids

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**Abstract**—Numerical analysis of local similarity solutions of laminar, free convective flow over a vertical plate with uniform wall temperature and surface heat flux cases in non-Newtonian power law fluids is considered. The governing boundary layer equations along with the boundary conditions are first cast into a dimensionless form by a pseudo-similarity variable transformation and the resulting system of equations is then solved by a finite difference method in conjunction with the cubic spline interpolation. Results are obtained for the cases of modified Prandtl numbers, based on the local value of  $x$ , of 1, 10, 100, and 1000 over a range of values of the fluid flow index from 0.5 to 1.5. Representative local Nusselt number and average Nusselt number as well as velocity and temperature profiles are presented. Comparisons with earlier works are also made for large modified Prandtl numbers.

## INTRODUCTION

DUE TO the importance of applications of non-Newtonian fluids in industries processing molten plastics, polymers, etc., considerable efforts have been conducted to understand the behaviour of non-Newtonian fluids. Heat transfer by free convection along a vertical plate has been analysed rather extensively in power law fluids. Acrivos [1] was the first to study the external laminar boundary layer equation using an asymptotic method to obtain the solutions which are simply appropriate to large modified Prandtl numbers. Subsequently, many works [2–12], including integral methods, experimental methods, and numerical methods, were also presented to yield the solutions of a vertical plate with uniform wall temperature and uniform surface heat flux conditions. Wang and Kleinstreuer [13] employed the Box method to solve a system of non-similar equations of a vertical plate immersed in power law fluids. However, only the isothermal cases of  $n = 0.891$  and 0.927 were presented. Lin [14] further proposed both the  $B$ -spline collocation method and the cubic spline collocation method to deal with the same problem, including both surface conditions. More recently, Shenoy and Mashelkar [15] and Irvine and Karni [16] made excellent reviews on the subject of convective heat transfer in non-Newtonian fluids.

In the present work the free laminar convective heat transfer from a vertical plate with uniform wall

temperature and surface heat flux to non-Newtonian power law fluids is reconsidered. A new appropriate pseudo-similarity variable transformation is proposed and the resulting transformed equations along with boundary conditions are very compact and simple. Numerical solutions are carried out for the transformed equations by an effective finite difference method together with the cubic spline interpolation scheme [17].

Numerical results of interest, such as the local and average Nusselt numbers, velocity distributions, and temperature distributions, are presented for a range of modified Prandtl numbers  $1 \leq Pr_x \leq 2000$  covering the fluid flow indices in the range of  $0.5 \leq n \leq 1.5$ .

## ANALYSIS

Consider a vertical flat plate situated in a quiescent bulk of non-Newtonian power law fluids with constant temperature  $T_\infty$ . The plate is prescribed with the uniform surface temperature (UST) or uniform surface heat flux (UHF). By employing the boundary layer model, Boussinesq approximations and the power law model for non-Newtonian fluids, the convective, laminar and steady conservation equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

## NOMENCLATURE

$f$	dimensionless stream function
$g$	gravitational acceleration
$h$	local heat transfer coefficient
$K$	fluid consistency index for power law fluid
$k$	thermal conductivity
$L$	reference length
$Nu_x$	local Nusselt number
$\bar{Nu}_L$	average Nusselt number
$n$	flow index for power law fluids
$Pr_x$	modified Prandtl number for UST case
$Pr_x^*$	modified Prandtl number for UHF case
$q_w$	wall heat flux
$Ra_x$	modified Rayleigh number for UST case
$Ra_x^*$	modified Rayleigh number for UHF case
$T$	fluid temperature
$u$	streamwise velocity
$v$	transverse velocity
$x$	streamwise coordinate
$y$	transverse coordinate.

## Greek symbols

$\alpha$	thermal diffusivity
$\beta$	coefficient of thermal expansion of fluid
$\delta$	velocity boundary layer thickness
$\delta_T$	thermal boundary layer thickness
$\eta$	pseudo-similarity variable
$\theta$	dimensionless temperature
$\rho$	density of fluid
$\psi$	stream function.

## Subscripts

$x$	local value in the $x$ -direction
$w$	wall condition
$\infty$	ambient condition.

## Superscripts

'	derivative, $d/d\eta$
*	uniform surface case
—	average value.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{K}{\rho} \frac{\partial}{\partial y} \left[ \left[ \frac{\partial u}{\partial y} \right]^{n-1} \frac{\partial u}{\partial y} \right] \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

The associated boundary conditions are

$$y = 0; \quad u = v = 0, \quad T = T_w \quad \text{or} \quad -k \frac{\partial T}{\partial y} = q_w$$

$$y \rightarrow \infty; \quad u \rightarrow 0, \quad T \rightarrow T_\infty$$

$$x = 0; \quad u = 0, \quad T = T_\infty. \quad (4)$$

First, using the scale analysis technique analogous to Newtonian fluids [18] in boundary layer equations (1)–(3), the thermal boundary layer thickness  $\delta_T(x)$  is written as

$$\delta_T(x) = \begin{cases} x/Ra_x^{1/(3n+1)} & \text{for UST case} \\ x/Ra_x^{*1/(3n+2)} & \text{for UHF case.} \end{cases} \quad (5)$$

Next, a pseudo-similarity variable  $\eta$ , a dimensionless stream function  $f(\eta)$ , and a dimensionless temperature  $\theta(\eta)$  are expressed respectively as

$$\eta = y/\delta_T(x), \quad f(\eta) = \psi(x, y)/(x\delta_T(x)) \quad (6)$$

and

$$\theta(\eta) = \begin{cases} \frac{T - T_\infty}{T_w - T_\infty} & \text{for UST case} \\ \frac{T - T_\infty}{q_w \delta_T(x)/k} & \text{for UHF case} \end{cases} \quad (7)$$

where the stream function defined by  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$  automatically satisfies continuity equation (1).

The governing equations (2) and (3) along with boundary conditions (4) can be transformed into the following system of equations:

$$a\{\theta + [f''']^{n-1} f'''\} + bff'' + cf'^2 = 0 \quad (8)$$

$$\theta'' + bff'\theta' + df'\theta = 0 \quad (9)$$

$$f(0) = f'(0) = 0, \quad \theta(0) = 1 \quad \text{or} \quad \theta'(0) = -1$$

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0. \quad (10)$$

where the primes denote differentiation with respect to  $\eta$ .  $a$ ,  $b$ ,  $c$ , and  $d$  are expressed respectively as

$$a = \begin{cases} \delta_T^4(x) g\beta(T_w - T_\infty)/(\alpha^2 x) & \text{for UST case} \\ \delta_T^5(x) g\beta q_w/(\alpha^2 x k) & \text{for UHF case} \end{cases}$$

$$b = 1 - \frac{x}{\delta_T} \frac{d\delta_T}{dx}$$

$$c = -\left(1 - \frac{2x}{\delta_T} \frac{d\delta_T}{dx}\right)$$

and

$$d = \begin{cases} 0 & \text{for UST case} \\ -\frac{x}{\delta_T} \frac{d\delta_T}{dx} & \text{for UHF case.} \end{cases}$$

For UST case,  $a = Pr_x^{2(n+1)/(3n+1)}$ ,  $b = (2n+1)/(3n+1)$ ,  $c = -(n+1)/(3n+1)$  and  $d = 0$  and for UHF case,  $a = Pr_x^{*(n+4)/(3n+2)}$ ,  $b = 2(n+1)/(3n+2)$ ,  $c = -(n+2)/(3n+2)$  and  $d = -n/(3n+2)$ . The modified Rayleigh number  $Ra_x$  ( $Ra_x^*$ ) and the modified Prandtl number  $Pr_x$  ( $Pr_x^*$ ) can be written as:

for UST case

$$Ra_x = \rho g \beta (T_w - T_\infty) x^{2n+1} / (K \alpha^n)$$

$$Pr_x = \frac{1}{\alpha} \left( \frac{K}{\rho} \right)^{2/(n+1)} x^{(n-1)/2(n+1)} \times [g \beta (T_w - T_\infty)]^{3(n-1)/2(n+1)}; \quad (11)$$

for UHF case

$$Ra_x^* = \rho g \beta q_w x^{2(n+1)} / (K \alpha^n k)$$

$$Pr_x^* = \frac{1}{\alpha} \left( \frac{K}{\rho} \right)^{5/(n+4)} x^{2(n-1)/(n+4)} [g \beta q_w / k]^{3(n-1)/(n+4)}. \quad (12)$$

The primary interest is the local Nusselt number,  $Nu_x = hx/k$ , which is readily written as

$$Nu_x / Ra_x^{1/(3n+1)} = -\theta'(0) \quad \text{for UST case} \quad (13)$$

$$Nu_x / Ra_x^{*1/(3n+2)} = 1/\theta(0) \quad \text{for UHF case} \quad (14)$$

and the average Nusselt number,  $\overline{Nu}_L = \overline{h}L/k$ , is

$$\overline{Nu}_L / Ra_L^{1/(3n+1)} = \frac{3n+1}{2n+1} [-\theta'(0)] \quad \text{for UST case} \quad (15)$$

$$\overline{Nu}_L / Ra_L^{*1/(3n+2)} = \frac{3n+2}{2(n+1)} [1/\theta(0)] \quad \text{for UHF case.} \quad (16)$$

## METHOD OF SOLUTION

The first step in solving the system of coupled, non-linear ordinary differential equations (8)–(10) is to convert equations (8) and (9) into a system of quasi-linear ordinary differential equations

$$A_0 f''' + A_1 f'' + A_2 f' + A_3 f + A_4 \theta = A_5 \quad (17)$$

$$B_0 \theta'' + B_1 \theta' + B_2 \theta + B_3 f' + B_4 f = B_5 \quad (18)$$

where

$$A_0 = an|f''|^{n-1}$$

$$A_1 = \pm an(n-1)|f''|^{n-2} f''' + bf$$

$$A_2 = 2cf'$$

$$A_3 = bf''$$

$$A_4 = a$$

$$A_5 = an(n-1)|f''|^{n-1} f''' + bf''' + cf'^2$$

$$B_0 = 1$$

$$B_1 = bf$$

$$B_2 = df'$$

$$B_3 = d\theta$$

$$B_4 = b\theta'$$

$$B_5 = bf\theta' + df''\theta.$$

Next, these quasi-linear differential equations (17) and (18) along with boundary conditions (10) are then cast into finite difference equations with the proper use of weighting factors. The resulting system of algebraic equations is then solved numerically by the Thomas method in conjunction with the cubic spline interpolation procedure [17]. This solution method has been found to yield rapid convergence and numerical results of high accuracy. It is very effective in dealing with the stiff equation (8) as the ratio of coefficients  $a/b$  becomes very large for high values of  $Pr_x$ . This numerical method shifts automatically from the central difference algorithm to the upwind difference algorithm, and vice versa. The details of this method are omitted here.

Since the thickness of boundary layer  $\eta_x$  depends on the modified Prandtl number  $Pr_x$ , the accuracy of the numerical results is checked by proceeding with the test of different mesh size  $\Delta\eta$  and thickness  $\eta_x$ . We use  $\Delta\eta = 0.01$  and  $\eta_x = 20$  as  $Pr_x \leq 10$ . For large values of  $Pr_x (\geq 100)$ ,  $\Delta\eta = 0.1$  and  $\eta_x = 40$  are adopted. It can be explained that equation (8) will be approximated by

$$\theta + [f''|f''|^{n-1}]' = 0 \quad (19)$$

with  $f(0) = f'(0) = f''(\infty) = 0$  and the numerical solutions are thus independent of  $Pr_x$ .

## RESULTS AND DISCUSSION

Free convection problems of non-Newtonian power law fluids including analytical and experimental results have been analysed since Acrivos [1]. For an isothermal vertical plate case, Fig. 1 depicts the effects of the modified Prandtl numbers on the local Nusselt number results in terms of  $Nu_x / Ra_x^{1/(3n+1)}$  as a function of the fluid flow index  $n$ . The integral solutions [3, 4] are also included in the figure. It can be seen from the figure that the local Nusselt number increases with increasing modified Prandtl numbers for all power law fluids. The local Nusselt number also increases monotonously as the value of the fluid flow index increases. In addition, for large values of  $Pr_x (\geq 100)$ , the local Nusselt number reaches a stationary value for all power law fluids. That is, as  $Pr_x \geq 100$ , equation (8) is approximated to equation (19) and thus its solution becomes independent of  $Pr_x$ . From Fig. 1, one can see that the local Nusselt number obtained by the integral method [4] compares well with that of the present work as  $n$  is larger than 0.9 and  $Pr_x \rightarrow \infty$ . Whereas, its value is higher than that of the present work as  $n < 0.9$ . Shenoy and Ulbrecht's result [3] deviates from that of the present work for all power law fluids even at high values of  $Pr_x$ . Consequently, in the present study consideration of the effects of modified Prandtl number defined as a function of  $x$ , ranging from 1 to 1000 is reasonable.

Representative dimensionless velocity and temperature profiles for values of  $n = 0.5, 1.0$ , and 1.5

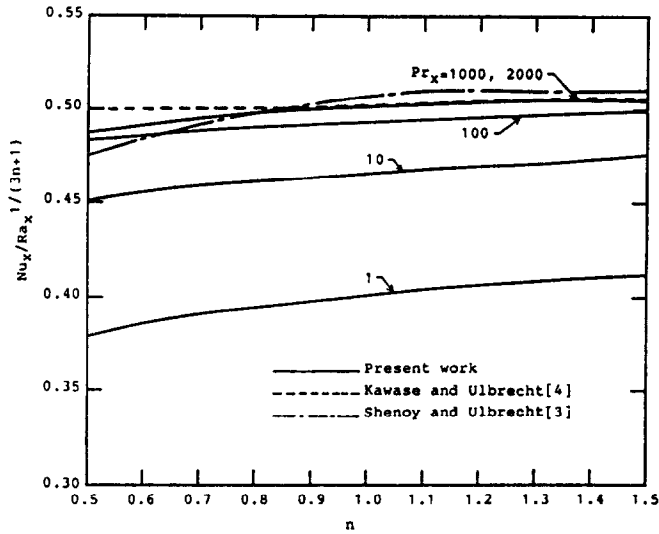


FIG. 1. Local Nusselt numbers in terms of fluid flow index for UST case.

and  $Pr_x = 1, 10, 100$ , and  $1000$  are illustrated, respectively, in Figs. 2 and 3. One can observe that the dimensionless velocity profiles are found to be strongly sensitive to the modified Prandtl number and the fluid flow index while the dimensionless temperature profiles are not obviously influenced. As

mentioned earlier, the ratio of thickness of momentum boundary layer and that of thermal boundary layer is

$$\frac{\delta(x)}{\delta_T(x)} \sim Pr_x^{2/(3n+1)} \quad (20)$$

It is obvious from Figs. 2 and 3 that for high values

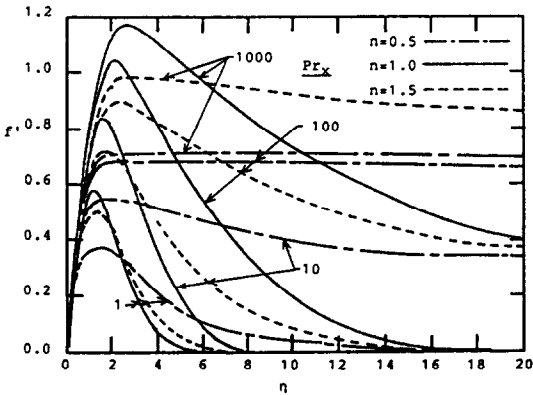


FIG. 2. Velocity profiles for UST case.

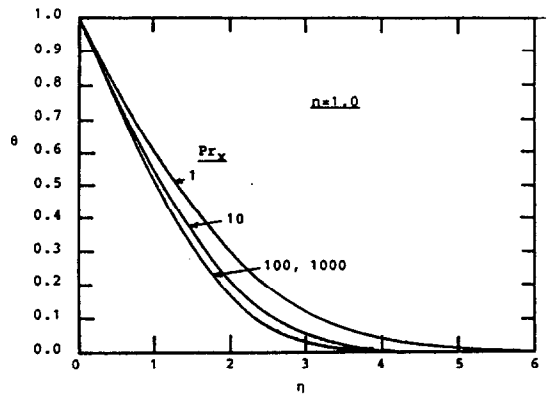


FIG. 3(b). Temperature profiles of  $n = 1.0$  for UST case.

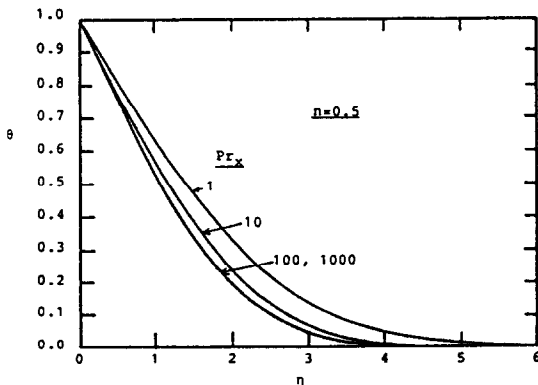


FIG. 3(a). Temperature profiles of  $n = 0.5$  for UST case.

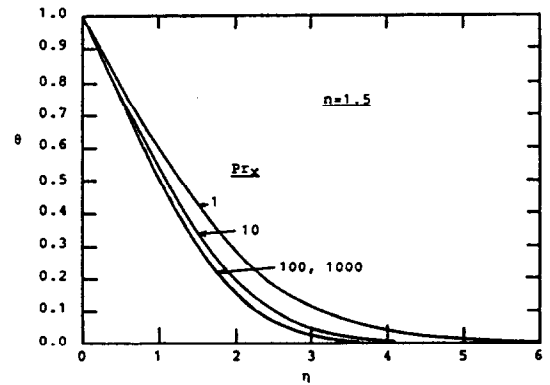


FIG. 3(c). Temperature profiles of  $n = 1.5$  for UST case.

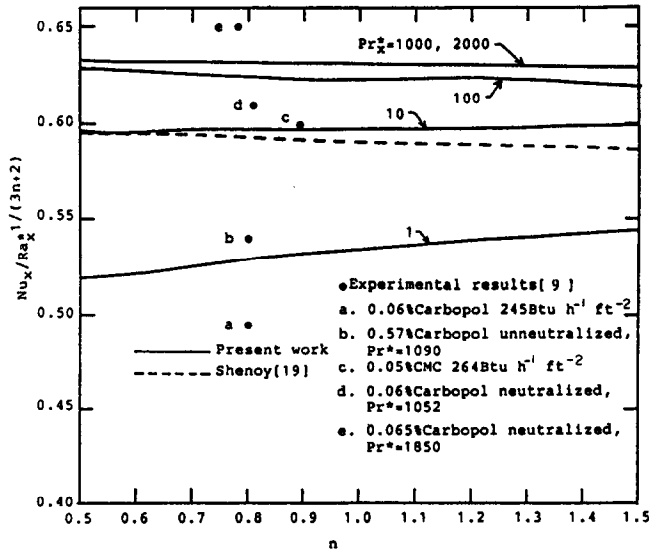


FIG. 4. Local Nusselt numbers in terms of fluid flow index for UHF case.

of  $Pr_x$ , the thinner the thermal boundary layer and the thicker the momentum boundary layer.

Figure 4 shows the laminar convective heat transfer from a vertical plate with uniform surface heat flux to power law fluids. It illustrates the local Nusselt

numbers as a function of the fluid flow index with a given value of  $Pr_x$ . Increasing the value of the modified Prandtl number, the local Nusselt number results in an increase in  $Nu_x / Ra_x^{1/(3n+2)}$  and reaches a uniform value. The figure also shows that the local Nusselt

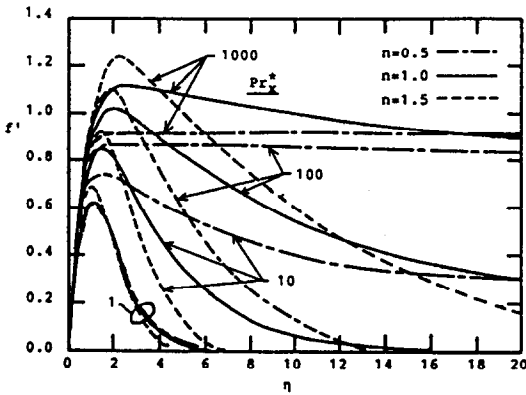


FIG. 5. Velocity profiles for UHF case.

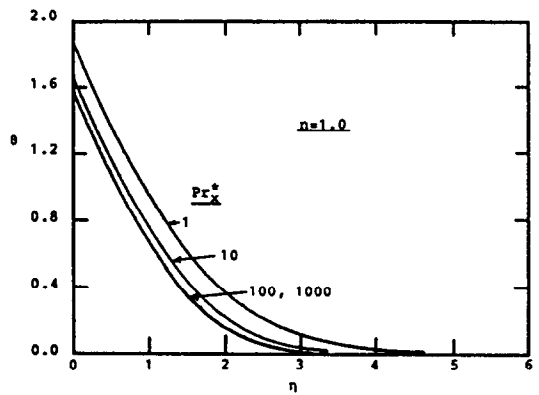


FIG. 6(b). Temperature profiles of  $n = 1.0$  for UHF case.

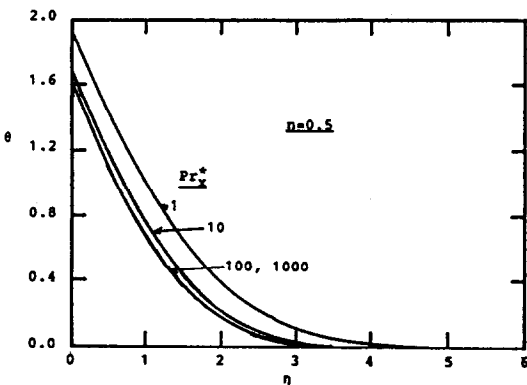


FIG. 6(a). Temperature profiles of  $n = 0.5$  for UHF case.

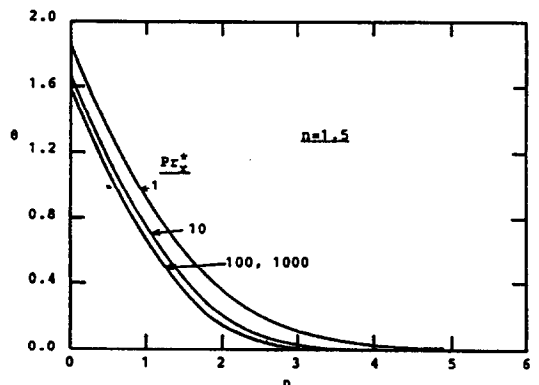


FIG. 6(c). Temperature profiles of  $n = 1.5$  for UHF case.

Table 1. Comparisons of average Nusselt number

(a) For isothermal case, $\overline{Nu}_L/Ra_L^{1/(3n+1)}$					
$n$	Shenoy and Ulbrecht [3]	Kawase and Ulbrecht [4]	Acrivos [1]	Tien [2]	Present work
0.5	0.5957	0.6275	0.63	0.6098	0.6105
1.0	0.6775	0.6700	0.67	0.6838	0.6701
1.5	0.7194	0.6960	0.71	0.7229	0.7012

(b) For constant heat flux case, $\overline{Nu}_L/Ra_L^{1/(3n+2)}$						
$n$	Chen and Wollersheim [10]	Lin [14]		Present work		
		$Pr_x^* = 100$	$Pr_x^* = 10$	$Pr_x^* = 2000$	$Pr_x^* = 100$	$Pr_x^* = 10$
0.5	0.7480	0.7293	0.7032	0.7381	0.7336	0.6907
1.0	0.7896	0.7725	0.7439	0.7883	0.7751	0.7445
1.5	0.8205	0.7995	0.7674	0.8213	0.8058	0.7790

number is insensitive to the fluid flow index and the same value of local Nusselt number for all power law fluids is found at  $Pr_x^* \rightarrow \infty$ . Som and Chen [12] proposed a correlation at  $Pr_x^*$

$$Nu_x/Ra_x^{1/(3n+2)} = 0.63. \quad (21)$$

This correlation is verified here. As can be seen from Fig. 4 the integral solution obtained by Shenoy [19] has a lower value than that of the present work. The experimental results [9] are also displayed in this figure.

Representative dimensionless velocity and temperature profiles for UHF case and for given values of  $Pr_x^*$  and  $n$  are illustrated in Figs. 5 and 6, respectively. Similarly, from the scale analysis

$$\frac{\delta(x)}{\delta_T(x)} \sim Pr_x^{*(n+4)/(n+1)(3n+2)}. \quad (22)$$

It is evident from Fig. 5 that the higher the value of  $Pr_x^*$  and the smaller the value of  $n$ , the thickness of the momentum boundary layer will become larger.

Table 1 simultaneously presents the average Nusselt numbers of these two surface conditions. The present average Nusselt number results are compared with those of earlier studies for distinct values of the fluid flow index and the modified Prandtl number. As can be seen from such a comparison, the agreement is found to be very good for high modified Prandtl numbers.

### CONCLUSIONS

A local similarity solution of laminar free convective heat transfer between a vertical flat plate and non-Newtonian power law fluids has been analysed. Both the isothermal wall case and the constant wall heat flux case have been considered. An appropriate coordinate transformation was yielded and an implicit finite difference scheme employed to the system of problems with the effects of the modified Prandtl num-

ber ranging from 1 to 2000 and the fluid flow index from 0.5 to 1.5. The results show that the local Nusselt number increases as the value of modified Prandtl number increases while the Nusselt number is less sensitive to the fluid flow index. The present results can be used to supplement earlier results of non-Newtonian power law fluids.

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#### SOLUTIONS LOCALEMENT AFFINES DE LA CONVECTION THERMIQUE NATURELLE SUR UNE PLAQUE VERTICALE POUR DES FLUIDES NON NEWTONIENS A LOI PUISSANCE

**Résumé**—On considère numériquement les solutions localement affines de la convection thermique laminaire sur un plan vertical à température ou à flux pariétal uniforme, dans le cas de fluides non newtoniens à loi puissance. Les equations de couche limite et les conditions aux limites sont mises sous une forme adimensionnelle par une transformation de variable pseudo-affine et le système d'équations résultant est résolu par une méthode aux différences finies, en relation avec l'interpolation spline cubique. Des résultats sont obtenus dans le cas des nombres de Prandtl modifiés, basés sur la valeur locale de  $x$ , égaux à 1, 10, 100 et 1000 et pour un domaine d'indice d'écoulement du fluide allant de 0,5 à 1,5. On présente les nombres de Nusselt locaux et globaux ainsi que les profils de vitesse et de température dans des cas typiques. On fait aussi la comparaison avec des travaux antérieurs pour des grands nombres de Prandtl modifiés.

#### ÖRTLICHE ÄHNLICHKEITSLÖSUNGEN FÜR DEN WÄRMEÜBERGANG DURCH FREIE KONVEKTION VON EINER VERTIKALEN PLATTE AN "POWER-LAW"- FLUIDE

**Zusammenfassung**—Es wird eine numerische Analyse von örtlichen Ähnlichkeitslösungen einer laminaren, frei konvektiven Strömung über eine vertikale Platte mit einheitlicher Wandtemperatur und einheitlichem Wärmestrom an der Oberfläche in nicht-Newton'schen "Power-Law"-Fluiden betrachtet. Die Grenzschicht-Gleichungen werden zuerst zusammen mit den Randbedingungen mit Hilfe einer Pseudo-Ähnlichkeitsvariablen-Transformation in eine dimensionslose Form gebracht; anschließend wird das resultierende Gleichungssystem mit einer Finite-Differenzen-Methode in Verbindung mit der kubischen Spline-Interpolation gelöst. Für die modifizierten Prandtl-Zahlen 1, 10, 100 und 1000, basierend auf örtlichen Werten von  $x$ , werden Ergebnisse ermittelt, und zwar für Fluid-Strömungs-Kennzahlen zwischen 0,5 und 1,5. Die repräsentative örtliche Nusselt-Zahl und die mittlere Nusselt-Zahl werden ebenso dargestellt wie die Geschwindigkeits- und die Temperaturprofile. Weiterhin werden Vergleiche mit früheren Arbeiten für große modifizierte Prandtl-Zahlen angestellt.

#### ЛОКАЛЬНЫЕ АВТОМОДЕЛЬНЫЕ РЕШЕНИЯ ЗАДАЧИ СВОБОДНОКОНВЕКТИВНОГО ТЕПЛОПЕРЕНОСА ОТ ВЕРТИКАЛЬНОЙ ПЛАСТИНЫ К НЕНЬЮТОНОВСКИМ СТЕПЕННЫМ ЖИДКОСТЯМ

**Аннотация**—Численно анализируются локальные автомодельные решения задачи ламинарного свободноконвективного течения над вертикальной пластиной в случае неньютоновских степенных жидкостей при постоянных температуре стенки и тепловом потоке на поверхности. Сначала определяющие уравнения пограничного слоя вместе с граничными условиями приводятся к безразмерной форме при помощи преобразования псевдоподобия, а затем полученная система уравнений решается конечно-разностным методом в комбинации с кубической сплайновой интерполяцией. Получены результаты для случаев модифицированных чисел Прандтля на основе локального значения  $x$ , равного 1, 10, 100 и 1000, в широком диапазоне значений индекса течения жидкости 0,5-1,5. Представлены характерные локальные и средние числа Нуссельта, а также профили скорости и температуры. Проведены сравнения с ранее опубликованными работами для случаев больших значений модифицированных чисел Прандтля.